DUAL SITESWAPS (v0.2)

Dual concerns Vanilla and Multiplex Siteswaps.

Dual of a Scramblable Siteswap

The Demonstration is straight:

1. Let's **S** a Vanilla Scramblable Siteswap of period p in the form $a_0 \ldots a_i \ldots a_{p-1}$. It then means that any rearrangement in the Siteswap gives a valid Siteswap $\mathbf{S_i}$
2. If we get the Dual with a maximum throw of height H, of every $\mathbf{S_i}$, we get a valid Siteswap. It then implies that any arrangement of $H-a_0, \ldots, H-a_i, \ldots, H-a_{p-1}$ gives a valid Siteswap.

⇨ **The Dual of a Vanilla Scramblable Siteswap is a Scramblable Siteswap.**

⇨ **We may use a very similar demonstration to show that The Dual of a Multiplex Scramblable Siteswap is a Multiplex Scramblable Siteswap.**

Dual of a Magic Siteswap

Strict Increasing Vanilla Magic Siteswap of Odd/Even parity

- Let's **S** a strict increasing Magic Siteswap of Odd/Even Parity of period p.
- S is thus in the form a_0 a_0+2 … a_0+2i … $a_0+2(p-1)$ with :
  - a_0 odd if Parity is Odd
  - a_0 even if Parity is Even
- The corresponding Dual with a maximum throw of height H is then:

  $H-a_0-2(p-1)$ … $H-a_0-2i$ … $H-a_0$

  This is a strict Increasing Magic Siteswap of period p and it's :
  - Odd if $H-a_0$ is Odd
  - Even if $H-a_0$ is Even

⇨ **The Dual of a Strict Increasing Vanilla Magic Siteswap of Odd/Even parity is a Strict Increasing Magic Siteswap :**

- **of Parity Odd if one of H or the originel Siteswap are Odd and the other is Even.**
- **of Parity Even if both H and the original Siteswap are Even or Odd**

If $H-a_0 == 2(p-1)$ (Even) or $H-a_0 == 2p-1$ (Odd), the Dual Magic Siteswap is complete. It contains all possible values.

Strict decreasing Vanilla Magic Siteswap of Odd/Even parity

- The demonstration is identical to the previous one. S is here in the form a_0 a_0-2 … a_0-2i … $a_0-2(p-1)$ with :
  - a_0 odd if Parity is Odd
  - a_0 even if Parity is Even

- The corresponding Dual with a maximum throw of height H is then:
  $H-a_0+2(p-1)$ … $H-a_0+2i$ … $H-a_0$
  This is a strict Decreasing Magic Siteswap of period p and it's :
  - Odd if $H-a_0$ is Odd
  - Even if $H-a_0$ is Even

⇨ **The Dual of a Strict Decreasing Vanilla Magic Siteswap of Odd/Even parity is a Strict Decreasing Magic Siteswap :**
  - **of Parity Odd if one of H or the originel Siteswap are Odd and the other is Even.**
  - **of Parity Even if both H and the original Siteswap are Even or Odd.**

  If $H-a_0 == 0$ (Even) or $H-a_0 == 1$ (Odd) , the Dual Magic Siteswap is complete. It contains all possible values.

Strict Increasing Vanilla Magic Siteswap without parity

- Let's **S** a strict increasing Magic Siteswap without Parity of period p.
- S is thus in the form a_0 a_0+1 … a_0+i … $a_0+(p-1)$
- The corresponding Dual with a maximum throw of height H is then:
  $H-a_0-(p-1)$ … $H-a_0-i$ … $H-a_0$
  This is a strict Increasing Magic Siteswap of period p

⇨ **The Dual of a Strict Increasing Vanilla Magic Siteswap without parity is a Strict Increasing Magic Siteswap.**
  If $H-a_0 == p-1$ (Even) or If $H-a_0 == p$ (Odd), the Dual Magic Siteswap is complete. It contains all possible values.

Strict Decreasing Vanilla Magic Siteswap without parity

- The demonstration is identical to the previous one. It gives:
- S is here in the form a_0 a_0-1 … a_0-i … $a_0-(p-1)$
- The corresponding Dual with a maximum throw of height H is then:
  $H-a_0+(p-1)$ … $H-a_0+i$ … $H-a_0$
  This is a strict Decreasing Magic Siteswap of period p

⇨ **The Dual of a Strict Decreasing Vanilla Magic Siteswap without parity is a Strict Decreasing Magic Siteswap.**
  If $H-a_0 == 0$ (Even) or If $H-a_0 == 1$ (Odd), the Dual Magic Siteswap is complete. It contains all possible values.

Vanilla Magic Siteswap without parity with split Odd/Even Throws in Strict Increasing/Decreasing Order

- We then have here a split between m Odd and n Even Throws (m+n=p). We just have to apply both previous results on each part to see that the Dual is also a Magic Siteswap in the same Ordering.

- **This is easy to find an example that shows that the Dual of a Multiplex Magic Siteswap is not necessarly a Magical Siteswap :**
  - 1[23]0 is a Multiplex Magic Siteswap.
  - Its Dual of maximum height 4 is [44][(4-3)(4-2)][(4-1)4] = [44][12][34] that is not a Magical Siteswap.

Dual of a Reversible Siteswap

The Demonstration is straight:

- Let's **S** a Reversible Vanilla Siteswap of period p in the form $a_0 \ldots a_i \ldots a_{p-1}$. It then means that S'=$a_{p-1} \ldots a_i \ldots a_0$ is also a valid Siteswap.
- If we get the Dual with a maximum throw of height H, of S we get a valid Siteswap in the form S_{dual} = H-a_{p-1}…H-a_i…H-a_0
- If we get the Dual with a maximum throw of height H, of S' we also get a valid Siteswap in the form S'$_{dual}$ = H-a_0…H-a_i…H-a_{p-1}
- Since S'$_{dual}$ is the Reversed of S_{dual} :

⇨ **The Dual of a Vanilla Reversible Siteswap is a Vanilla Reversible Siteswap**

⇨ **We may use a very similar demonstration to show that The Dual of a Multiplex Reversible Siteswap is a Multiplex Reversible Siteswap.**

Dual of a Palindrome Siteswap

- **A Palindrome is a particular Reversible Siteswap.**
- Let's **S** a Vanilla Palindrome Siteswap of period p in the form $a_0a_1 \ldots a_i \ldots a_1a_0$.
  The Dual with a maximum throw of height H is then S'=H-a_0 H-a_1…H-a_i … H-a_1 H-a_0 that is also valid and a Palindrome.

⇨ **The Dual of a Vanilla Palindrome Siteswap is a Vanilla Palindrome Siteswap.**

⇨ **We may use a very similar demonstration to show that The Dual of a Multiplex Palindrome Siteswap is a Multiplex Palindrome Siteswap.**

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